Embeddings of cartesian products of two regular graphs

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The cartesian product of two cycles always has an orientable embedding for which all faces are quadrilaterals, on the torus. In 1992 Pisanski asked whether the product of two \( r \)-regular graphs always has an orientable quadrilateral embedding, in particular when the two graphs are 1-factorable. We provide examples to show that the answer to Pisanski’s question is negative. We also describe some situations where an orientable quadrilateral embedding does exist. The positive results can be described using a new construction that gives some other general results.

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