Dot Product Graphs: Investigation of Extreme Dimensions

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A *dot product graph* is a graph G such that there exists a function $f : V(G) \to I\!R^k$ such that for $x \cdot y \in V(G)$, $xy \in E(G)$ if and only if $f(x)^T f(y) \geq 1$. The minimum k such that there exists such a function f for G is the *dot product dimension of* G. It was conjectured that the maximum dot product dimension of a graph on n vertices is $\left\lfloor \frac{n}{2} \right\rfloor$.
In this talk, we explore graphs of dot product dimension 1 and graphs with maximum dot product dimension. We will give a correction to the forbidden induced subgraph characterization of graphs with dot product dimension of 1. Also, we will introduce a new, possibly fruitful, approach to proving the conjectured bound of $\left\lfloor \frac{n}{2} \right\rfloor$; in particular, the edge cover of a graph by 1-dot product graphs.

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