Matroidal Structure of Skew Polynomial Root Sets

Travis Baumbaugh*, Felice Manganiello, Clemson University

A skew polynomial ring $R = K[x; \sigma, \delta]$ is a ring of polynomials with coefficients in a field K with the standard addition but multiplication defined by the rule $xa = \sigma(a)x + \delta(a)$ for any $a \in K$. For such polynomials, left and right divisibility differ, and thus any polynomial $f(x) \in R$ has left and right roots, which may differ. Since such polynomials may also have more right or left roots than their degree, we have the concept of independence of roots. If the smallest polynomial to have every element of $Z \subset K$ as right roots has degree $|Z|$, then Z is right independent, and likewise for left independence. These right and left independent sets define matroids over K.
It is these matroids we are interested in. If we take $R = \mathbb{F}_{q^m}[x, \sigma]$ ($\delta \equiv 0$), then there is a natural bijection between the independent sets of these two matroids.

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