

Compositions colored by simplicial polytopic numbers

Daniel Birmajer, Nazareth College of Rochester

For a given integer $d \geq 1$, we consider $\binom{n+d-1}{d}$ -color compositions of a positive integer ν for which each part of size n admits $\binom{n+d-1}{d}$ colors. We give explicit formulas for the enumeration of such compositions, generalizing existing results for n -color compositions (case $d = 1$) and $\binom{n+1}{2}$ -color compositions (case $d = 2$). In addition, we give bijections from the set of $\binom{n+d-1}{d}$ -color compositions of ν to the set of compositions of $(d+1)\nu - 1$ having only parts of size 1 and $d+1$, the set of compositions of $(d+1)\nu$ having only parts of size congruent to 1 modulo $d+1$, and the set of compositions of $(d+1)\nu + d$ having no parts of size less than $d+1$. Our results rely on basic properties of partial Bell polynomials and on a suitable adaptation of known bijections for n -color compositions.

This is joint work with Juan Gil and Michael Weiner.

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