Geometric Representation of Graphs and Crossing Numbers

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The *distance* $d_G(u, v)$ of two vertices u and v in a connected graph G is the number of edges in a shortest u-v-path in G. We say that a connected graph G can be embedded into the euclidean plane with *distortion* at most α if there is an injection f of the vertex $V(G)$ into the euclidean plane such that for any two vertices $u, v \in V(G)$

$$\frac{1}{\alpha} \leq \frac{||f(u) - f(v)||}{d_G(u, v)} \leq \alpha.$$

The main result of the talk is that for any $\alpha \geq 1$ and any Δ there is a number $h(\alpha, \Delta)$ such that every graph with maximum degree Δ that can be embedded into the euclidean plane with distortion at most α has a plane drawing with at most $h(\alpha, \Delta)$ crossings per edge.

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