An overview of pattern avoiding permutations Something old, something new

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We say that a permutation p contains the shorter permutation q as a pattern if p contains $|q|$ entries, not necessarily in consecutive positions, whose pairwise relations to each other are the same as those of the entries of q. For instance, $p = 3576241$ contains $q = 231$, since the first, third and fifth entries of p relate to each other as the entries of q, namely the leftmost entry is the second smallest, the middle one is the largest, and the rightmost entry is the smallest.

In the first part of this talk, we will review the early results of this fascinating and rapidly growing topic, including the celebrated Marcus-Tardos theorem from 2003. That theorem shows that for any given pattern q, the number of permutations of length n that avoid q is simply exponential, that is, there exists a constant c_q so that $S_n(q) \leq c_q^n$.

In the second part, we discuss some more recent developments, such as a sequence of results on the extremely tenacious pattern 1324, a surprising connection to stack-sortable permutations, and the disproof of numerous long-standing conjectures. Many open problems will also be discussed.