

Monochromatic Rectangles in Grid Colorings

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Ramsey Theory is a branch of combinatorics in which structures like graphs, grids, and integers are colored, and interesting properties are proven to arise. We focus on colorings of 2-dimensional grids and prove certain properties regarding shapes such as monochromatic rectangles. For example, every coloring of the vertices of a 5 by 5 grid using 2 colors must contain a monochromatic rectangle with sides parallel to the x and y axes (a previously known result). We find better upper bounds on the size of grids that must contain monochromatic rectangles and L shapes (3 specific points of a rectangle) with specified dimensions, culminating with a generalization of the Square Theorem. We address lower bounds on grids containing rectangles with specific dimensions, which has never before been addressed, and provide a state of the art proof technique. Finally, we analyze grids that avoid all monochromatic rectangles, including non axis-parallel rectangles, and find new and strict bounds.

Keywords: Ramsey theory, grid, monochromatic, coloring, rectangle