Enumerating Hamiltonian Paths in Cayley Digraphs on the Semidirect Product of Cyclic Groups

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Let $\epsilon_1, \epsilon_2 \in \{-1, 1\}$. Let m and n be odd positive integers such that $n = \epsilon_2 \pmod 4$. Let G be the semidirect product of cyclic groups of order $8m$ and $2n$ given by $G = \mathbb{Z}_{8m} \rtimes \mathbb{Z}_{2n} = \langle x, y : x^{8m} = 1,\ y^{2n} = 1,\ \text{and}\ yxy^{-1} = x^{2m\epsilon_1 - 1} \rangle$. The Cayley digraph on G with generating set $\{x, y\}$, denoted by $\text{Cay}(G : x, y)$, is the digraph with vertex set G and arc set consisting of arcs from g to gs whenever $g \in G$ and $s \in \{x, y\}$. We determine a formula for the number of hamiltonian paths in $\text{Cay}(G : x, y)$ (with initial vertex 1) that depends on the parameters m, n, ϵ_1 and ϵ_2. In addition, we determine all hamiltonian paths in $\text{Cay}(G : x, y)$ (with initial vertex 1).

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