PROJECT TITLE

Constructions of hypergraphs free of certain 3-uniform 3-partite hypergraphs

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INTRODUCTION

Early proofs of the lower bounds for $ex(n;K_{2,2})$ and $ex(n;K_{3,3})$ relied on finite geometries; finite projective planes for $ex(n;K_{2,2})$ (see Reiman 1959 [1]) and Brown's construction of $K_{3,3}$-free graphs using finite spheres [2]. For 3-partite hypergraphs, the lower bounds of $(c + o(1))n^{8/3}$ for $ex(n; K^{(3)}(2,2,3))$ and $ex(n;K^{(3)}(2,2,2))$ were proved by Mubayi [3] and Katz, Kropp and Magioni [4] respectively. Mubayi's construction is a generalization of projective norm-graphs constructed by Alon, Rónyai, and Szabó [5], while Katz, Kropp, and Magioni use algebra to construct affine planes that intersect four in at most a point. The goal of this research is to reprove the lower bounds for $ex(n; K^{(3)}(2,2,3))$ and $ex(n;K^{(3)}(2,2,2))$ using geometry, by generalising Brown's construction to $K^{(3)}(2,2,3)$-free hypergraphs, and by using cross products to construct planes that intersect four in at most a single point.

SIGNIFICANCE

The construction of a $K^{(3)}(2,2,3)$-free hypergraph highlights how to generalise a result for the extremal number of a bipartite graphs to that of a 3-uniform 3-partite hypergraph, while the construction of a $K^{(3)}(2,2,2)$-free hypergraph introduces cross products as a way to construct graphs that attain the best known lower bound for $ex(n;K^{(3)}(2,2,2))$.

STUDENT INVOLVEMENT

This is my own work.

REFERENCES

[1] I. Reimann, Uber ein Problem von K. Zarankiewicz, *Acta Math. Acad. Sci. Hung.* **9** (1959), 269-279.

[2] W. G. Brown, On graphs that do not contain a Thomsen graph, *Canad. Math. Bull.* **8** (1966), 281-285.

[3] D. Mubayi, Some exact results and new asymptotics for hypergraph Turán numbers, *Combin., Prob. Comp.* **11** (2002), 299-309.

[4] N. H. Katz, E. Krop, and M. Maggioni, Remarks on the box problem *Math. Res. Lett.* **9** (2002), 515-519.

[5] N. Alon, L. Rónyai, and T. Szabó, Norm-graphs: variations and applications, *J. Combin. Theory Ser. B* **76** (1999), 280-290.