

Polynomial Lemniscate Configurations and Lemniscate Trees

Michael Epstein*, Erik Lundberg, Florida Atlantic University

We consider the combinatorial class of labeled, nonplanar, rooted trees in which vertices have outdegree at most two, such that the labels increase along any directed path in the tree. We refer to these as *lemniscate trees* since this class arises as the solution to an enumeration problem involving topological equivalence classes of generic polynomial lemniscate configurations. With the goal of establishing a baseline for studies of nonlocal statistics in random polynomials, we investigate the typical shape of a lemniscate tree sampled uniformly at random from the combinatorial class. We apply complex analytic methods to a bivariate generating function in order to determine (asymptotically in the size of the tree) the mean and variance of the number of vertices of outdegree two.

Keywords: polynomial lemniscates, lemniscate trees, analytic combinatorics