Using Matching to Detect Infeasibility of Some Integer Programs

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We present a novel heuristic matching-based algorithm intended to detect specially formulated infeasible IPs. It either detects an infeasible IP or exits undecided. It does not solve an IP. We call it *the triple overlay matching based closure algorithm* (the algorithm). Input to the algorithm is a $\{0,1\}$ IP whose constraints are a set of nested doubly stochastic subsystems together with a set of instance defining variables set at zero level. Its solution set is a subset of the set of $n!$ nxn permutation matrices P, written as $n!$ n^2xn^2 block permutation matrices Q each with block structure P. Output from the algorithm is a certificate of infeasibility, or an undecided IP and a set of variables deduced to be at zero level. Infeasible IPs may fail to be detected infeasible, while feasible IPs must fall in the undecided category. We present an application for the algorithm, a specially constructed $\{0,1\}$ IP model of the Hamilton tour decision problem, including empirical results. We successfully apply the algorithm to over 2,100 non-Hamiltonian graphs. No IPs fail that are not reported. We show how to model both the graph and subgraph isomorphism decision problems for input to the algorithm. We propose the algorithm can be developed for use with current solvers i.e. its output might help direct where and how to search for possible solutions.

Keywords: integer program, matching, permutation, decision problem