

Up-Down Ternary Strings

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Let Σ be the alphabet $\{0, 1, 2\}$, ordered as $0 < 1 < 2$. For $n \geq 2$, an element $x_1 x_2 \dots x_n \in \Sigma^n$ is called an *up-down* (ternary) string of length n when $x_1 < x_2$, $x_2 > x_3$, \dots , and $x_{n-1} < x_n$, for n even, while $x_{n-1} > x_n$, for n odd. If $n = 4$, for instance, there are eight such ternary strings, namely

0101, 0102, 0212, 0201, 0202, 1212, 1201, 1202.

In general, the number of such strings of length n is given by F_{n+2} , the $(n+2)$ nd Fibonacci number. Among the statistics for these F_{n+2} strings of length n , we determine

(1) the number of occurrences of each of 0, 1, and 2; (2) the sum of these strings considered as base 3 integers; and, (3) the number of inversions that occur.

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