Embedding Complete Multipartite Graphs into Smallest Dimension

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For a finite, simple graph G, define G to be of dimension d if d is the minimum value such that G can be drawn with vertices being points of $\mathbb{R}^d$ where adjacent vertices are necessarily placed a unit distance apart. We determine the dimension of all complete multipartite graphs. Letting G be a complete multipartite graph with n parts, m of which have size one or two, our main result is that G is of dimension $2n - m - 1$ when all parts or all but one part of G has size one, and G is of dimension $2n - m$ otherwise.

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