

## Oriented Graceful Labelings of Trees

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For 50 years, the Graceful Tree Labeling Conjecture has withstood significant efforts to either prove the result or find a counterexample. We introduce a stronger conjecture in which we posit that for every tree  $T$  with bipartition  $V(T) = A \cup B$ , there exists a graceful labeling  $f : V(T) \rightarrow \{0, 1, 2, \dots, n - 1\}$  with the additional property that for every vertex  $a \in A$ ,  $f(a) < f(b)$  for every  $b \in N(a)$ . We call labelings with this property *oriented* because if we orient each edge in  $T$  so that it points from  $A$  to  $B$ ,  $f(a) < f(b)$  will hold for every arc  $ab$ , but not necessarily for nonadjacent  $a \in A$  and  $b \in B$ . We discuss motivation for this conjecture, some special classes for which is true, some computational results, and directions for further research.

Keywords: graceful labeling, Graceful Tree Labeling Conjecture