

On 2-fold graceful labelings of graphs

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Let \mathbb{Z} denote the set of integers and \mathbb{N} denote the set of nonnegative integers. For integers a and b with $a \leq b$, let $[a, b] = \{x \in \mathbb{Z} : a \leq x \leq b\}$. For a positive integer k , let 2K_k denote the 2-fold complete multigraph of order k . Similarly, let ${}^2[a, b]$ denote the multiset that contains every element of $[a, b]$ exactly two times. Let G be a multigraph of size n , order at most $n + 1$, and edge multiplicity at most 2. A *labeling* of G is a one-to-one function $f: V(G) \rightarrow \mathbb{N}$. If f is a labeling of G and $e = \{u, v\} \in E(G)$, let $\bar{f}(e) = |f(u) - f(v)|$. A *2-fold graceful labeling* of G is a one-to-one function $f: V(G) \rightarrow [0, n]$ such that:

$$\{\bar{f}(e) : e \in E(G)\} = \begin{cases} {}^2[1, \frac{n}{2}] & \text{if } n \text{ is even,} \\ {}^2[1, \frac{n-1}{2}] \cup \{\frac{n+1}{2}\} & \text{if } n \text{ is odd.} \end{cases}$$

A graph G is said to be *2-fold graceful* if it admits a *2-fold graceful labeling*. It is known that if G with n edges is 2-fold graceful, then there exists a cyclic G -decomposition of ${}^2K_{n+1}$. It is conjectured that every tree is 2-fold graceful. We investigate 2-fold graceful labelings of various classes of graphs including several classes of trees. This work was completed as part of the *Illinois State University REU for Pre-service and In-service Secondary Mathematics Teachers*.

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