

Cubic surfaces over \mathbb{F}_{13}

Fatma Karaoglu*, James W. P. Hirschfeld, University of Sussex

Given five skew lines a_1, a_2, a_3, a_4, a_5 with a single transversal b_6 such that each set of four a_i omitting a_j ($j = 1, \dots, 5$) has a unique further transversal b_j , then the five lines b_1, b_2, b_3, b_4, b_5 also have a transversal a_6 . These twelve lines form a double-six. The double six lies on a unique cubic surface with 15 further lines c_{ij} given by $[a_i, b_j] \cap [a_j, b_i]$.

The necessary and sufficient condition for the existence of a double six, and so of a cubic surface with 27 line, is the existence in a plane over the same field of a 6-arc not on a conic. We introduce a geometric procedure for the classification of cubic surfaces arising from a double-six over the finite field \mathbb{F}_q of characteristic other than two. The performance of the algorithm is illustrated by the example of cubic surfaces over \mathbb{F}_{13} .

Keywords: double-six, 6-arc, Eckardt point