Bounds for the Sum Choice Number

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Let $G = (V(G), E(G))$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$, and for every vertex $v \in V(G)$ let $L(v)$ be a set (list) of available colors. The graph G is called *L-colorable* if there is a proper coloring ϕ of the vertices with $\phi(v) \in L(v)$ for all $v \in V(G)$. A function f from the vertex set $V(G)$ of G to the positive integers is called a *choice function* of G and G is said to be *f-list colorable* if G is L-colorable for every list assignment L with $|L(v)| = f(v)$ for all $v \in V(G)$. Set $\text{size}(f) = \sum_{v \in V(G)} f(v)$ and define the *sum choice number* $\chi_{sc}(G)$ as minimum of size(f) over all choice functions f of G.

It is easy to see that $\chi_{sc}(G) \leq |V(G)| + |E(G)|$ for every graph G and that there is a greedy coloring of the vertices of G for the corresponding choice function f and every list assignment L with $|L(v)| = f(v)$ for all $v \in V(G)$.

Obviously, if $\chi_{sc}(G) \leq k$ and H is a subgraph of G, then $\chi_{sc}(H) \leq k$. Therefore, this property is a hereditary graph property. This implies $\chi_{sc}(G) \geq 2|V(G)| - 1$ for a connected graph G since $\chi_{sc}(T) = 2|V(G)| - 1$ for a spanning tree T of G.

In this talk we will improve the above mentioned upper and lower bounds for the sum choice number. We will present several general lower and upper bounds on $\chi_{sc}(G)$ in terms of subgraphs of G.

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