

# Edge Colorings of Hypergraphs Without A Fixed Rainbow Colored Subhypergraph

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We consider a hypergraph version of a graph-theoretical problem that has been proposed by Erdős and Rothschild in 1974, and which has been studied by several authors in the last few years: for a fixed number  $r \geq 2$  of colors and a fixed forbidden graph  $F$ , they were interested in those  $n$ -vertex graphs that allow the largest number of  $r$ -edge-colorings avoiding a monochromatic copy of  $F$ . Yuster, for  $F$  being a triangle  $K_3$  and  $r = 2$  colors, and later Alon, Balogh, Keevash and Sudakov showed that, for  $r \in \{2, 3\}$  colors and  $n$  large, the  $n$ -vertex  $\ell$ -partite Turán graph admits the largest number of colorings avoiding a monochromatic copy of the complete graph  $K_{\ell+1}$ . For  $r \geq 4$  colors the situation changes.

Very recently a version of this problem for colorings avoiding *rainbow* copies of graphs  $F$ , namely copies of  $F$  in which every edge is colored differently, has been investigated.

Here, we focus on rainbow hyperedge-colorings of hypergraphs. For  $k$ -uniform hypergraphs  $F$  and  $H$ , and an integer  $r$ , let  $c_{r,F}(H)$  be the number of  $r$ -colorings of the set of hyperedges of  $H$  with no rainbow copy of  $F$ . Let  $c_{r,F}(n) = \max_{H \in \mathcal{H}_{n,k}} c_{r,F}(H)$ , where the maximum runs over the family  $\mathcal{H}_{n,k}$  of all  $k$ -uniform hypergraphs on  $n$  vertices. As usual,  $ex(n, F)$  denotes the Turán number for  $F$ .

Every  $r$ -coloring of an  $F$ -free hypergraph  $H$  contains no rainbow copy of  $F$ , and consequently  $c_{r,F}(n) \geq r^{ex(n,F)}$  for all  $r \geq 2$ . It turns out that, for some classes of forbidden linear, uniform hypergraphs, we indeed have  $c_{r,F}(n) = r^{ex(n,F)}$  for all fixed  $r \geq r_0$  and  $n$  large, namely for the 3-uniform hypergraph Fano plane, and the two families of  $k$ -uniform expanded, complete graphs and Fan( $k$ )-hypergraphs. Moreover, the extremal  $n$ -vertex hypergraphs turn out to be unique and are isomorphic to the Turán hypergraph for  $F$ .

Keywords: edge colorings, Turán problem, extremal hypergraphs