

Online coloring blowups of a known graph

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In the G -coloring game of width w , both G and w are known to the two players, who alternate turns. First, Spoiler places a new token at a vertex in G , and Algorithm responds by assigning a color to the new token. Algorithm must ensure that tokens on the same or adjacent vertices receive distinct colors. Spoiler must ensure that the token conflict graph (in which two tokens are adjacent if and only if their distance in G is at most 1) has chromatic number at most w . Algorithm wants to minimize the number of colors used, and Spoiler wants to force as many colors as possible. The *value* of the G -coloring game of width w , denoted $f_G(w)$, is the minimum number of colors needed in an optimal Algorithm strategy. A graph G is *online-perfect* if $f_G(w) = w$. We give a forbidden induced subgraph characterization of the class of online-perfect graphs. When G is not online-perfect, determining $f_G(w)$ seems challenging; we establish $f_G(w)$ asymptotically for some (but not all) of the minimal graphs that are not online-perfect. The game is motivated by a natural online coloring problem on the real line which remains wide open.

Keywords: online graph coloring, combinatorial game