Degree complete graphs

Sebastian Milz, RWTH Aachen University

Let $G = (V, E)$ be a graph with an ordered vertex set $V = \{1, \ldots, n\}$. We call a vector of nonnegative integers $S = (s_1, \ldots, s_n)$ a degree vector of G if there is an orientation D of G such that $s_i = d_D^+(i)$ for all $i \in V$. It is known that every degree vector satisfies

$$S_G^l \preceq S \preceq S_G^r, \quad \sum_{i=1}^{n} s_i = |E| \quad \text{and} \quad 0 \leq s_i \leq d_G(i) \quad \text{for all} \quad i = 1, \ldots, n, \qquad (1)$$

where S_G^l (S_G^r) is the minimal (maximal) degree vector of G with respect to the domination order. The graph G is called degree complete if every vector s satisfying condition (1) is a degree vector of G. In 2006 Qian characterized degree complete graphs with ordered vertex sets.

Suppose we are given a graph G without an ordered vertex set Qian asked how to decide whether there is an ordering of the vertices of G that yields a degree complete graph. Answering this problem we give two characterizations of the class of graphs which have such a degree complete vertex set ordering. Moreover we state a polynomial procedure to find a desired ordering of the vertices of G if it exsists.

Keywords: degree vector, degree complete