Graph Dimension and a Quick Extremal Problem

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For a finite, simple graph G, say G is of dimension n, and write $\dim(G) = n$, if n is the smallest integer such that G can be represented with vertices being points of $\mathbb{R}^n$ and adjacent vertices necessarily being placed a unit-distance apart. A question attributed to Paul Erdős, and described by Alexander Soifer in *The Mathematical Coloring Book*, asks for the minimum number of edges in a graph of dimension 4. In a 2013 article, R. F. House showed the answer to this question is 9 with this minimum being realized uniquely by $K_{3,3}$. In this talk, we give a new and very short proof of this result – not a one-liner mind you, but more like a four- or five-liner (supposing of course that several of those lines are run-on sentences). We close by showing how our method extends to answer the question in dimensions 5 and 6 as well.

Keywords: graph dimension, unit-distance graph embeddings