

Search games and optimal Kakeya sets

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A planar set that contains a unit segment in every direction is called a Kakeya set. These sets have been studied intensively in measure theory and harmonic analysis since Besicovich (1919); we find a new connection to combinatorics and game theory. A hunter and a rabbit move on an n -vertex cycle without seeing each other until they meet. At each step, the hunter moves to a neighboring vertex or stays in place, while the rabbit is free to jump to any node. Thus they are engaged in a zero sum game, where the payoff is the capture time. We show that every rabbit strategy yields a Kakeya set; the optimal rabbit strategy is based on a Cauchy random walk, and it yields a Kakeya set K consisting of $4n$ triangles, that has minimal area among such Kakeya sets. I'll conclude with an open problem: is the capture time of a weak rabbit (that can only jump to distance 1) on a general n -vertex graph, linear in n ? (Talk based on joint work with Y. Babichenko, R. Peretz, P. Sousi and P. Winkler).