

Special Divisors of $n^k + 1$

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For a positive integer k , we ask:

For which $n > 0$ does $n^k + 1$ have a divisor congruent to $-1 \pmod{n}$?

We classify such n in the cases $k = 1, 2, 3, 4$. Utilizing the notion of k -chains, we show that there are infinitely many such n in the cases $k > 4$. We also illustrate a connection with a famous problem from the International Mathematics Olympiad, and provide some generalizations.

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