Efficient Domination

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Let $G\Box H$ denote the Cartesian product of G with H. For each $h \in V(H)$, $G\Box\{h\}$ is a copy of G called a G-layer (or G-fiber) of $G\Box H$. Similarly, for each $g \in V(G)$, $\{g\}\Box H$ is a copy of H called a H-layer (or H-fiber). An efficient dominating set in $G\Box H$ is a dominating set $D \subseteq V(G\Box H)$ with $|N[v] \cap D| = 1$ for all $v \in V(G\Box H)$.
A minimum dominating set $D \subseteq V(G\Box H)$ is G-layer (Cartesian product) efficient if for all $v \in \{\langle G\Box\{h\}\rangle - N[D|_{G\Box\{h\}}]\}$, we have $|N[v] \cap D| = 1$, for all $h \in V(H)$. That is, all vertices not dominated by vertices in their own G-layer are dominated exactly once by a vertex in a different G-layer. By definition, any efficient dominating set is both G-layer and H-layer efficient, however, the converse need not be true. We present two natural generalizations of layer efficiency to include graphs that are not representable as Cartesian products. These generalizations are extended to several variants of domination.

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