Towards the Reconstruction of Decomposable Ordered Sets?

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The **reconstruction problem** for ordered sets asks if (the isomorphism type of) every ordered set P with $|P| \geq 4$ can be reconstructed from its collection (or "deck") $\langle P \setminus \{x\} : x \in P\rangle$ of unlabeled one-point-deleted ordered subsets. An ordered set is called **decomposable** iff there is a subset $A \subseteq P$ so that $1 < |A| < |P|$ and so that, for all $x \in P \setminus A$, we have that, if $x > a$ for some $a \in A$, then $x > a$ for all $a \in A$, and, if $x < a$ for some $a \in A$, then $x < a$ for all $a \in A$. Subsets A as just described are called **nontrivial order-autonomous** subsets.
This talk will provide useful tools for the reconstruction of many, but not all, decomposable ordered sets. One key insight is that reconstruction attempts will split naturally between ordered sets with ≥ 3 elements in nontrivial order-autonomous subsets and ordered sets with exactly 2 elements in nontrivial order-autonomous subsets. The culminating result is that decomposable ordered sets with canonical decomposition $P = L\{P_t | t \in T\}$ so that there is a $t \in T$ with $|P_t| > 1$ and so that, for $t \neq s$, $T \setminus \{t\}$ is not isomorphic to $T \setminus \{s\}$ are reconstructible.

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