On Balance Index Sets of k-level Wheel Graphs

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Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$, and let $\mathbb{Z}_2 = \{0, 1\}$. A labeling $f : V(G) \to \mathbb{Z}_2$ induces an edge partial labeling $f^* : E(G) \to \mathbb{Z}_2$ defined by $f^*(uv) = f(u)$ if and only if $f(u) = f(v)$ for each edge $uv \in E(G)$. For $i \in \mathbb{Z}_2$, let $v_f(i) = |\{v \in V(G) : f(v) = i\}|$ and $e_f(i) = |\{e \in E(G) : f^*(e) = i\}|$. We call f is a friendly labeling if $|v_f(0) - v_f(1)| \leq 1$. The balance index set of G, denoted BI(G), is defined as $\{|e_f(0) - e_f(1)| :$ the vertex labeling f is friendly.$\}$. A k-level wheel can be constructed recursively as follows: a wheel is a 1-level wheel and the cycle of the wheel is the 1-level cycle. A k-level wheel is obtained from a $(k-1)$-level wheel by appending an edge from any number of vertices of the $(k-1)$-level cycle to one new vertex which form the vertices in the k-level cycle. In this paper, we study the balance index sets of the of k-level wheels.

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