On the solution space of the Golomb recursion

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We explore the nature of the solution space for the Golomb nested recursion $g(n) = g(n - g(n-1))+1$. On this solution space, we define a natural equivalence relation and restrict our attention to non-equivalent solutions. We describe and prove an algorithm that determines whether a given set of initial conditions generates a solution. Up to equivalence, there is a unique solution whose forward differences are eventually either 0 or 1, namely, the Golomb sequence $g_0 = 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots$, generated by the initial condition $g_0(1) = 1$. This sequence is asymptotic to $\sqrt{2n}$; we conjecture that this is true of every solution. We further conjecture that each solution has what we call a generational structure that abstracts combinatorial properties of g_0. It appears that for any given solution, its generations are composed of only a finite number of building blocks.

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