Arcs, Caps and Codes: Old Results, New Results, Generalizations

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A non-singular conic of the projective plane PG(2, q) over the finite field GF(q) consists of q+1 points no three of which are collinear. For q odd, this non-collinearity condition for q+1 points is sufficient for them to be a conic; see Segre (1954). Generalizing, Segre considers sets of k points in PG(2, q), $k \geq 3$, no three of which are collinear, and also sets of k points in the n-dimensional projective space PG(n, q) over GF(q), $k \geq n+1$, no n+1 of which lie in a hyperplane; the latter are k-arcs. There is a close relationship between k-arcs and certain algebraic curves, hypersurfaces of PG(n, q), and linear MDS codes.
The concept of a k-arc in PG(2, q) was generalized to that of a k-cap in PG(n, q); a k-cap of PG(n, q), $n \geq 3$, is a set of k points no three of which are collinear. An elliptic quadric of PG(3, q) is a cap of size q^2+1. In 1955, Barlotti and Panella independently showed that, for q odd, the converse is true. Also, q^2+1 is the maximum size of a k-cap in PG(3, q) for $q \neq 2$. This leads to the definition of an ovoid of PG(3, q) as a cap of size q^2+1 for $q \neq 2$ and, for q = 2, a cap of size 5 with no 4 points in a plane. Ovoids of particular interest were discovered by Tits (1962).
Arcs and caps can be generalized by replacing their points with m-dimensional subspaces to obtain generalized k-arcs and generalized k-caps.
The talk will be a survey on arcs, caps, generalized arcs and generalized ovoids; it will also contain recent results and open problems.

Keywords: finite projective spaces, arcs, caps, ovals, ovoids, MDS codes