The Game of Thrones

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The game of thrones is a two-player impartial combinatorial game played on a tournament (an oriented complete graph). Suppose $T = (V, A)$ is a tournament. A *king* in T is a vertex k such that for any other vertex x, either $(k, x) \in A$ or there exists a vertex y such that $(k, y) \in A$ and $(y, x) \in A$. An *heir* in T is a vertex that is not a king but becomes a king when some vertex is deleted from T. Players take turns deleting vertices from a given tournament until there are no heirs. The winning player is the one which makes the final move. We develop winning positions and a strategy for play that is independent of the structure of the tournament for certain tournaments on which the game begins. We develop algorithms for winning on some other tournaments, but the problem of finding a winning strategy in general is not solved.

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