Stability of the potential function

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A graphic sequence π is potentially H-graphic if there is some realization of π that contains H as a subgraph. The Erdős-Jacobson-Lehel problem [Graphs realizing the same degree sequence and their respective clique numbers. Graph Theory, Combinatorics and Applications (eds. Alavi, Chartrand, Oellerman and Schwenk), Vol. 1, 1991, 439–449] asks to determine σ(H, n), the minimum even integer such that any n-term graphic sequence π with sum at least σ(H, n) is potentially H-graphic. The parameter σ(H, n) is known as the potential function of H, and can be viewed as a degree sequence variant of the classical extremal function ex(n, H). Recently, Ferrara, LeSaulnier, Moffatt and Wenger [On the sum necessary to ensure that a degree sequence is potentially H-graphic, to appear in Combinatorica] determined σ(H, n) asymptotically for all H, which is a potential function analogue to the Erdős-Stone-Simonovits Theorem.

In this talk, we investigate a stability concept for the potential number, inspired by Simonovits’ [A method for solving extremal problems in graph theory, stability problems. In: Theory of Graphs, Proc. Colloq., Tihany, 1966, Academic Press, New York, 1968, pp. 279–319] classical result on the stability of the extremal function. Simonovits’ theorem demonstrates that for every nonbipartite graph H with chromatic number r + 1, any H free graph with close to ex(n, H) edges must have structure approximated by that of the Turán graph T_{n,r}. We first define a notion of stability for the potential number that is a natural analogue to the stability given by Simonovits, and demonstrate that not all graphs are σ-stable. We then give a sufficient condition for a graph H to be stable with respect to the potential function, and characterize the stability of those graphs H that contain an induced subgraph of order α(H) + 1 with exactly one edge.

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