Matchings in Hypergraphs

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A matching in a hypergraph is a set of disjoint edges. It is a well-known difficult problem to give good lower bounds on the maximum size of a matching in a hypergraph in terms of other natural parameters. Here we discuss tools for this, with a focus on the special case of tripartite hypergraphs: those for which the vertex set can be partitioned into three parts, such that each edge contains exactly one vertex from each part. For example, if a tripartite hypergraph is $r$-regular (meaning that each vertex is in exactly $r$ edges) with $n$ vertices in each class then it has a matching of size at least $n/2$, and this is tight for certain special hypergraphs. We investigate how this bound can be improved for all other hypergraphs.

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