Spectrum of the Szlam Numbers of the Plane

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Suppose that the plane $\mathbb{R}^2$ is equipped with a translation invariant distance function $\rho$ and suppose that $d > 0$. The distance graph $G_\rho(\mathbb{R}^2, d)$ is the graph with vertex set $\mathbb{R}^2$ with $u, v \in \mathbb{R}^2$ adjacent if and only if $\rho(u, v) = d$. A rather red coloring of $G$ is a coloring of $\mathbb{R}^2$ with red and blue such that no two points adjacent in $G$ are both blue. The Szlam number of $G$ is the minimum cardinality, over all rather red colorings of $G$, of $X \subseteq \mathbb{R}^2$ such that no translate of $X$ is all red. Fixing $d = 1$, we exploit results of Johnson, Szlam, and Kloeckner to show that for every positive integer $n$ there exists $\rho$ such that the Szlam number of $G_\rho(\mathbb{R}^2, 1)$ is $n$.

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