Choosability with Union Separation

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List coloring generalizes graph coloring by requiring the color of a vertex to be selected from a list of colors specific to that vertex. One refinement of list coloring, called choosability with separation, requires that the intersection of adjacent lists is sufficiently small. We introduce a new refinement, called choosability with union separation, where we require that the union of adjacent lists is sufficiently large. For \( t \geq k \), a \((k, t)\)-list assignment is a list assignment \( L \) where \( |L(v)| \geq k \) for all vertices \( v \) and \( |L(u) \cup L(v)| \geq t \) for all edges \( uv \). A graph is \((k, t)\)-choosable if there is a proper coloring for every \((k, t)\)-list assignment. We explore this concept through examples of graphs that are not \((k, t)\)-choosable, demonstrating sparsity conditions that imply a graph is \((k, t)\)-choosable, and proving that all planar graphs are \((3, 11)\)-choosable and \((4, 9)\)-choosable.

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