Some Properties of Acyclic Heaps of Pieces
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Let $W$ be an arbitrary Coxeter group with generating set $S$ of involutions. A reduced expression for an element $w \in W$ is a minimal length word in $S$ that represents $w$. The set $W_c$ of fully commutative elements are characterized by the property that any reduced expression for $w \in W_c$ can be obtained from any other via iterated commutations of adjacent generators. If $w \in W_c$ and $sw \notin W_c$ for some $s \in S$, then we say $sw$ is weakly complex.

Star reducible Coxeter groups are a class of Coxeter groups whose fully commutative elements have a particularly nice property. Star reducible Coxeter groups contain the finite Coxeter groups as a subclass.

A heap is an isomorphism class of labelled posets. Each heap is equipped with a set of edges and a set of vertices. Green defined a linear map $\partial_E$ which sends each edge of a heap $E$ to a linear combination of vertices. If $v \in \text{Im}\partial_E$, we call $v$ a boundary vertex. If $e_0$ is an edge of $E$ and $\partial_E(e_0) = v$ for a vertex $v$, then we call $v$ an effective boundary vertex. If $\partial_E(e_0) = v_1 + v_2$, then $v_1$ and $v_2$ are said to be linearly equivalent. A result by Stembridge states that every fully commutative element has a unique heap.

The main result we will prove here is that in the heap of a fully commutative element in a star reducible Coxeter group, every boundary vertex is linearly equivalent to an effective boundary vertex.