Prime labelings of generalized Petersen graphs and large cubic bipartite graphs

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A graph $G$ is called prime if there exists a labeling of the vertices of $G$ with distinct labels $1, 2, ..., |V(G)|$ such that the labels on any two adjacent vertices are relatively prime; such a labeling is called a prime labeling. In general, bipartiteness is neither necessary nor sufficient for a graph to be prime. S. Schluchter, J.Z. Schroeder, T. Wilson, et. al. conjectured that a generalized Petersen graph $P(n, k)$ is prime if and only if it is bipartite, which occurs for $n$ even and $k$ odd, and they verified that the conjecture holds for all even $n \leq 50$ and all possible odd $k$. We give more evidence to support this conjecture. In this talk we will develop a prime labeling algorithm that has verified the conjecture for all even $n$ and odd $k$ such that $2 \leq n \leq 9000$ and all possible odd $k$. We also develop a different prime labeling method that we have used to prime label all cubic bipartite graphs $G$ such that $8 \leq |V(G)| \leq 22$.

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